2-3 有理式函數之極限

1. \[ \lim_{x \to 1} \left(1 - \frac{x - 1}{x^2 + 2x - 3}\right) = ? \quad (A) -\infty \quad (B) -\frac{1}{4} \quad (C) 0 \quad (D) \frac{3}{4} \]

解答：(D)

Sol：
\[ \lim_{x \to 1} \left(1 - \frac{x - 1}{x^2 + 2x - 3}\right) = \lim_{x \to 1} \frac{x - 1}{x^2 + 2x - 3} = \frac{1}{4} - \frac{3}{4} = \frac{3}{4} \quad \# \]

2. 求 \[ \lim_{x \to 2} \left(\frac{1}{2x^2 + 3x - 14} - \frac{1}{3x^2 - x - 10}\right) \]

解答：\[ \frac{1}{121} \]

Sol：
\[ \lim_{x \to 2} \left(\frac{1}{2x^2 + 3x - 14} - \frac{1}{3x^2 - x - 10}\right) = \lim_{x \to 2} \left(\frac{1}{(2x + 7)(x - 2)} - \frac{1}{(3x + 5)(x - 2)}\right) \]

通分
\[ \lim_{x \to 2} \left(\frac{3x + 5 - 2x - 7}{(2x + 7)(x - 2)(3x + 5)}\right) = \lim_{x \to 2} \left(\frac{x - 2}{(2x + 7)(x - 2)(3x + 5)}\right) \]

\[ = \lim_{x \to 2} \frac{1}{(2x + 7)(3x + 5)} = \frac{1}{121} \quad \# \]

3. 已知 \[ \lim_{x \to 2} \frac{x^2 + ax + b}{x^2 - x - 2} = 2 \]，求 a, b 之值。

解答：a = 2, b = -8

Sol：\[ \lim_{x \to 2} \frac{x^2 + ax + b}{x^2 - x - 2} = 2 \Rightarrow \text{为} \quad \frac{0}{0} \text{型的不定型極限} \]

\[ \Rightarrow 2^2 + 2a + b = 0, \quad 2a + b = -4 \]

使用羅必達法則，得
\[ \lim_{x \to 2} \frac{x^2 + ax + b}{x^2 - x - 2} = \lim_{x \to 2} \frac{2x + a}{2x - 1} = \frac{4 + a}{3} = 2 \]

\[ \Rightarrow 4 + a = 6, \quad a = 2 \quad \text{代回(1)式得} \quad b = -8 \quad \# \]